ORDER STATISTICS FROM INVERSE RAYLEIGH DISTRIBUTION

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Abstract

This paper develops the single and product moments of order statistics from inverse Rayleigh (IR) and doubly truncated IR distributions with some recurrence relations. Finally we use some of these relations to characterize the IR distribution.

Keywords: Inverse Rayleigh (IR); order statistics; truncated IR distributions.

1. INTRODUCTION

Order statistics arise naturally in many real-life applications involving data relating to life testing studies. The order statistics and its moments have assumed considerable interest in recent years; see for example, David (1981), Arnold et al. (1992) and Balakrishnan and Cohan (1991), Pickands (1975) and Childs et al. (2000) have derived the higher-order moments of order statistics from power function distribution and Pareto distributions, respectively. For extensive survey of moments of order statistics, see Balakrishnan and Sultan (1998) have studied order statistics and its properties from exponential distribution. Balakrishnan and Chan (1998) have discussed linear estimation for log-gamma parameters based on order statistics. Balakrishnan and Aggarwala (1998) have obtained the single and double moments of order statistics from generalized logistic distribution. They also applied their results to draw inference. Masoom and Umbach (1998) have drawn optimal linear inference by using selected order statistics in location-scale model. Khan et al. (1983) and Ali et al. (1998) have established some recurrence relations for the single and product moments. Voda (1972) proposed the following function as a model in reliability theory

$$R(x) = 1 - e^{-\alpha / x^{\beta}} \qquad \alpha, x, \beta > 0,$$

when he take $\beta = 2$ and $\alpha = \theta$, obtain the reliability function, distribution and probability density functions of the inverse Rayleigh distribution which given by,

$$R(x) = 1 - e^{-\theta / x^2},$$

$$F(x) = e^{-\theta / x^2}, \qquad (1.1)$$

and

$$f(x) = \frac{2\theta}{x^3} e^{-\theta / x^2} \qquad \theta, x > 0 \qquad (1.2)$$

from (1.1) and (1.2), we have

$$f(x) = \frac{2\theta}{x^3} F(x),$$

Voda (1972) presented properties of the maximum likelihood estimator $\hat{\theta}$ of the parameter θ of the inverse Rayleigh distribution. Gharrph (1993) made comparisons of estimators of location measures of the inverse Rayleigh distribution with some applications in the reliability studies. The inverse Rayleigh distribution provides a good fit to several life data.

Let $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$ be the order statistics of a sample of size n from IR distribution given in (1.2). Then, the probability density function of the r^th order statistic $x_{(r)}$, say $x_{:n}$, $r = 1, 2, \ldots, n$ is given by David (1981)

$$f_{r,n}(x) = \frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1 - F(x)]^{n-r} f(x), \quad -\infty < x < \infty$$

$$= \frac{n!}{(r-1)!(n-r)!} \frac{2\theta}{x^3} \; e^{-r\theta / x^2} \left[1 - e^{-\theta / x^2}\right]^{n-r}, \qquad x > 0 \qquad (1.3)$$

The joint density function of $x_{(r)}$ and $x_{(s)}$ $(1 \leq r < s \leq n)$ is given by David (1981)

$$f_{r,s;n}(x,y) = \frac{n!}{(r-1)!(s-r-1)!(n-s)!} [F(x)]^{r-1}$$

$$[F(y) - F(x)]^{s-r-1} [1 - F(y)]^{n-s} f(x) f(y), \quad -\infty < x < y < \infty$$

$$= \frac{n! \, e^{-\theta / y^2}}{(r-1)!(s-r-1)!(n-s)!} \frac{4\theta^2}{x^3 y^3} e^{-r\theta / x^2}$$

$$\left(e^{-\theta / y^2} - e^{-\theta / x^2}\right)^{s-r-1} \left(1 - e^{-\theta / y^2}\right)^{n-s}, \quad 0 < x < y < \infty \qquad (1.4)$$

The doubly truncated IR distribution may be used to describe some phenomena that depend on time start from $t_0 \neq 0$. The probability density function of the doubly truncated IR distribution may be written from (1.2) as

$$g(x) = \frac{2\theta / x^3}{U - V} e^{-\theta / x^2}, \qquad v < x < u, \quad \theta > 0 \qquad (1.5)$$

where

$$U = e^{-\theta / u_1^2} \quad and \quad V = e^{-\theta / v_1^2}, \qquad (1.6)$$

with distribution function given by

$$G(x) = \frac{1}{U - V} \left(e^{-\theta / x^2} - V\right), \qquad (1.7)$$

where U *and* V are given in (1.6), then from (1.5) and (1.7), we have

$$g(x) = \frac{2\theta}{x^3}(G(x) + V), \tag{1.8}$$

Then the probability density function of the r[th] order statistic in a sample from the doubly truncated IR distribution may be written as

$$g_{r,n}(x) = \frac{n!}{(r-1)!(n-r)!}[G(x)]^{r-1}[1-G(x)]^{n-r}\, g(x)$$

$$= \frac{n!}{(r-1)!(n-r)!}\frac{2\theta/x^3}{(U-V)^n} e^{-\theta/x^2}\left(e^{-\theta/x^2} - V\right)^{r-1}\left(U - e^{-\theta/x^2}\right)^{n-r}, \quad u < x < v \tag{1.9}$$

The joint probability density function of $x_{(r)}$ and $x_{(s)}$ $(1 \le r < s \le n)$ is given by

$$g_{r,s,n}(x,y) = \frac{n!}{(r-1)!(s-r-1)!(n-s)!}\frac{4\theta^2}{x^3 y^3}\cdot\frac{e^{-\theta/x^2}e^{-\theta/y^2}}{(U-V)^n}$$

$$\left(e^{-\theta/x^2} - V\right)^{r-1}\left(e^{-\theta/y^2} - e^{-\theta/x^2}\right)^{s-r-1}\left(U - e^{-\theta/y^2}\right)^{n-s}, \quad v \le x \le y \le u \tag{1.10}$$

In section (2) we establish some new recurrence relations for the single and product moments of order statistics in the non-truncated and truncated cases; we use some of these relations to give conclusion and remarks in section 3.

2. MOMENTS OF ORDER STATISTICS

In this section, we establish some new recurrence relations for the single and product moments of order statistics from IR distribution. By using (1.3), the single k[th] moment $\mu_{r:n}^{(k)}$, $(1 \le r \le n,\ k = 0,1,2,...)$ is given by

$$\mu_{r:n}^{(k)} = \frac{n!}{(r-1)!}\frac{2\theta}{(n-r)!}\int_0^\infty x^{k-3}\, e^{-r\theta/x^2}\left(1 - e^{-\theta/x^2}\right)^{n-r} dx,$$

while the single k[th] moment $\mu_{r:n}^{(k)}$, $(1 \le r \le n,\ k = 0,1,2,...)$ of order statistics from the doubly truncated IR distribution using (1.9) is given by

$$\mu_{r:n}^{(k)} = \frac{n!}{(r-1)!(n-r)!}\frac{2\theta}{(U-V)^n}\int_v^u x^{k-3}\, e^{-\theta/x^2}\left(e^{-\theta/x^2} - V\right)^{r-1}\left(U - e^{-\theta/x^2}\right)^{n-r} dx \tag{2.1}$$

where U *and* V are given in (1,6).

Similarly the product $(k_1, k_2)^{th}$ moment $\mu_{r,s:n}^{(k_1,k_2)}$ of $x_{(r)}$ and $x_{(s)}$, $k_1, k_2 = 0, 1, \ldots$, $1 \le r < s \le n$ may be written from (1.4) as

$$\mu_{r,s:n}^{(k_1,k_2)} = \frac{n!\,(4\theta^2)}{(r-1)!(s-r-1)!(n-s)!}\int_0^\infty\int_x^\infty x^{k_1-3} y^{k_2-3} e^{-\theta/y^2}\, e^{-r\theta/x^2}$$

$$\left(e^{-\theta/y^2} - e^{-\theta/x^2}\right)^{s-r-1}\left(1 - e^{-\theta/y^2}\right)^{n-s} dy dx \tag{2.2}$$

while the product moment $\mu_{r,s;n}^{(k_1,k_2)}$ of $x_{(r)}$ and $x_{(s)}$ from doubly truncated IR distribution is given using (1.10) by

$$\mu_{r,s,n}^{(k_1,k_2)} = \frac{n!\,4\theta^2}{(r-1)!(s-r-1)!(n-s)!}\frac{1}{(U-V)^n}\int_v^u\int_x^u x^{k_1-3}y^{k_2-3}e^{-\theta/x^2}e^{-\theta/y^2}$$

$$\left(e^{-\theta/x^2}-V\right)^{r-1}\left(e^{-\theta/y^2}-e^{-\theta/x^2}\right)^{s-r-1}\left(U-e^{-\theta/y^2}\right)^{n-s}dydx \qquad (2.3)$$

2.1 Recurrence relations based on the non-truncated case

2.1.1. Single moments

<u>Theorem 1</u>

The single moment $\mu_{r;n}^{(k)}$, $k > 2$, $1 \le r \le n-1$ satisfies the following relation,

$$\mu_{r;n}^{(k)} = \frac{2\theta}{k-2}\left[(n-r)\mu_{r+1;n}^{(k-2)} - r\mu_{r;n}^{(k-2)}\right] \quad . \qquad (2.4)$$

<u>Proof</u>

$$\mu_{r;n}^{(k)} = A\int_0^\infty x^{k-3}e^{-r\theta/x^2}\left(1-e^{-\theta/x^2}\right)^{n-r}dx$$

where $A = 2\theta\cdot\dfrac{n!}{(r-1)!(n-r)!}$.

By using integration by parts. We have

$$\mu_{r;n}^{(k)} = \frac{A}{k-2}\left[2\theta(n-r)\int_0^\infty x^{(k-3)-2}e^{-(r+1)\theta/x^2}\left(1-e^{-\theta/x^2}\right)^{n-(r+1)}dx\right.$$

$$\left. - 2\theta\, r\int_0^\infty x^{(k-3)-2}e^{-r\theta/x^2}\left(1-e^{-\theta/x^2}\right)^{n-r}dx\right]$$

$$\mu_{r;n}^{(k)} = \frac{2\theta}{k-2}\left[(n-r)\mu_{r+1;n}^{(k-2)} - r\,\mu_{r;n}^{(k-2)}\right]$$

2.1.2 Product moments

By using (2.2) and following the same technique used by Mohie El-Din et al. (1997), it can be seen that the product moment $\mu_{r,s;n}^{(k_1,k_2)}$ satisfies the following relation

<u>Theorem 2</u>

For $1 \le r < n-2$, $k_2 > 2$ and $k_1, k_2 > 0$ we have

$$\mu_{r,r+1;n}^{(k_1,k_2)} = \frac{2\theta}{k_2-2}\left[n\,\mu_{r,r+1;n-1}^{(k_1,k_2-2)} - (n-r)\,\mu_{r,r+1;n}^{(k_1,k_2-2)} - r\,\mu_{r+1;n}^{(k_1+k_2-2)}\right] \qquad (2.5)$$

Proof

From (2.2) we have

$$\mu_{r,s,n}^{(k_1,k_2)} = \frac{n!\,4\theta^2}{(r-1)!(s-r-1)!(n-s)!}\int_0^\infty x^{k_1-3}\,e^{-r\theta/x^2}\,I(x)\,dx \qquad (2.6)$$

where

$$I(x) = \int_x^\infty y^{k_2-3}e^{-\theta/y^2}\left(e^{-\theta/y^2} - e^{-\theta/x^2}\right)^{s-r-1}\left(1-e^{-\theta/y^2}\right)^{n-s} dy \qquad (2.7)$$

then, for $s = r+1$ in (2.7), we get

$$\begin{aligned} I(x) &= \int_x^\infty y^{k_2-3}e^{-\theta/y^2}\left(1-e^{-\theta/y^2}\right)^{n-r-1} dy \\ &= I_1 - I_2, \end{aligned} \qquad (2.8)$$

where

$$I_1 = \int_x^\infty y^{k_2-3}\left(1-e^{-\theta/y^2}\right)^{n-r-1} dy \qquad (2.10)$$

and

$$I_2 = \int_x^\infty y^{k_2-3}\left(1-e^{-\theta/y^2}\right)^{n-r} dy \qquad (2.11)$$

By using integration by parts, we have

$$\begin{aligned} I_1 &= \frac{-x^{k_2-2}}{k_2-2}\left(1-e^{-\theta/x^2}\right)^{n-r-1} \\ &\quad + \frac{2\theta(n-r-1)}{k_2-1}\int_x^\infty y^{(k_2-3)-2}e^{-\theta/y^2}\left(1-e^{-\theta/y^2}\right)^{n-r-2} dy \end{aligned} \qquad (2.9)$$

Similarly

$$\begin{aligned} I_2 &= \frac{-x^{k_2-2}}{k_2-2}\left(1-e^{-\theta/x^2}\right)^{n-r} \\ &\quad + \frac{2\theta(n-r)}{k_2-2}\int_x^\infty y^{(k_2-3)-2}e^{-\theta/y^2}\left(1-e^{-\theta/y^2}\right)^{n-r-1} dy \end{aligned} \qquad (2.10)$$

Substituting (2.9) and (2.10) into (2.8) we have

$$I(x)=\frac{1}{k_2-1}\Big[-x^{k_2-2}e^{-\theta/x^2}\left(1-e^{-\theta/x^2}\right)^{n-r-1}$$

$$+\ 2\theta(n-r-1)\int_x^\infty y^{(k_2-3)-2}e^{-\theta/y^2}\left(1-e^{-\theta/y^2}\right)^{n-r-2}dy$$

$$+\ 2\theta(n-r)\int_x^\infty y^{(k_2-3)-2}e^{-\theta/y^2}\left(1-e^{-\theta/y^2}\right)^{n-r-1}dy\Big] \qquad (2.11)$$

Substituting (2.11) into (2.6), we have

$$\mu_{r,r+1;n}^{(k_1,k_2)}=\frac{n!}{(n-r-1)(r-1)!}\cdot\frac{4\theta^2}{k_2-2}$$

$$*\Big[(n-r-1)\int_0^\infty\int_x^\infty x^{(k_1-3)}y^{(k_2-3)-2}e^{-\theta/y^2}e^{-r\theta/x^2}(1-e^{-\theta/y^2})^{n-r-1}dydx$$

$$-(n-r)\int_0^\infty\int_x^\infty x^{k_1-3}y^{k_2-3-2}e^{-\theta/y^2}e^{-r\theta/x^2}\left(1-e^{-\theta/y^2}\right)^{n-r-1}dydx$$

$$-\int_0^\infty x^{(k_1-3)+(k_2-3)-2}\,e^{-(r+1)\theta/x^2}\left(1-e^{-\theta/x^2}\right)^{n-r-1}dx\Big]$$

By simplifying the last equation we obtain the relation (2.5).

Result

For $1\le r<3\le n-1$, $n-3\ge 2$, $k_2>2$ and $k_1, k_2>0$ we have

$$\mu_{r,s;n}^{(k_1,k_2)}=\frac{1}{k_2-2}\left[n\{\mu_{r,s;n-1}^{(k_1,k_2-2)}-\mu_{r,s-1;n-1}^{(k_1,k_2-2)}\}-(n-s-1)\{\mu_{r,s;n}^{(k_1,k_2-2)}-\mu_{r,s-1;n}^{(k_1,k_2-2)}\}\right] \qquad (2.12)$$

2.2 Recurrence relations based on the doubly truncated case

2.2.1 Single moments

From (2.1), we note that the single moment $\mu_{r,n}^{(k)}$, k = 0, 1, 2, . . . satisfies the following relation

Theorem 3

For $u\le x_{(r),n}\le v$ and $k>2$ we have

$$\mu_{1,n}^{(k)}=\frac{2n\theta}{k-2}\left[\frac{U}{U-V}\mu_{1;n-1}^{(k-2)}-\frac{V}{U-V}v^{(k-2)}-\mu_{1;n}^{(k-2)}\right] \qquad (2.13)$$

where U *and* V are given by (1.6).

Proof

By putting r = 1 in (2.1), we have

$$\mu_{1;n}^{(k)} = \frac{2n\theta}{(U-V)} \int_v^u x^{k-3} e^{-\theta/x^2} \left(U - e^{-\theta/x^2}\right)^{n-1} dx \tag{2.14}$$

From (1.5) to (1.8) and (2.16), we have

$$\begin{aligned} \mu_{1;n}^{(k)} &= 2n\theta \int_v^u x^{k-3} (1 - G(x))^{n-1} (G(x) + V)\, dx \\ &= 2n\,\theta\left[I_1 + I_2\right] \end{aligned} \tag{2.15}$$

where

$$I_1 = V \int_v^u x^{k-3} [1 - G(x)]^{n-1} dx$$

and

$$I_2 = \int_0^u x^{k-3} G(x) [1 - G(x)]^{n-1} dx$$

By integration by parts, then we have

$$I_1 = \frac{V}{(U-V)} \frac{1}{k-2} \left[-u^{k-2} + (n-1) \int_v^u x^{k-2} g(x) [1 - G(x)]^{n-2} dx \right]. \tag{2.16}$$

Similarly

$$I_2 = \frac{1}{k-2} \left[(n-1) \int_v^u x^{k-2} g(x) [1 - G(x)]^{n-2} dx - - n \int_v^u x^{k-2} g(x) [1 - G(x)]^{n-1} dx \right]. \tag{2.17}$$

Substituting (2.16) and (2.17) into (2.15) we obtain

$$\begin{aligned} \mu_{1,n}^{(k)} = \frac{2n\theta}{k-2} \Bigg[&-v^{k-2} \frac{V}{U-V} + (n-1) \frac{U}{U-V} \int_v^u x^{k-2} g(x) [1 - G(x)]^{n-2} dx \\ &- n \int_v^u x^{k-2} g(x) [1 - G(x)]^{n-1} dx \Bigg] \end{aligned} \tag{2.18}$$

by simplifying (2.18), relation (2.13) is proved.

2.2.2 Product moments

From (2.3) we can see that, the product moment $\mu_{r,s;n}^{(k_1,k_2)}$ satisfies the follow theorem.

Theorem 4

For $1 \le r \le n-2$, $k_2 > 2$ and $k_1, k_2 > 0$, we have

$$\mu_{r,r+1;n}^{(k_1,k_2)} = \frac{2\theta}{k_2-2}\left[n\frac{V}{U-V}\mu_{r,r+1,n-1}^{(k_1,k_2-2)} - (n-r)\mu_{r,r+1,n}^{(k_1,k_2-2)} - r\mu_{r+1;n}^{(k_1+k_2-2)} - nv\mu_{r;n-1}^{(k_1+k_2-2)}\right] \tag{2.19}$$

where U *and* V are given by (1.6).

Proof

Put $s = r+1$ in (2.3) we have

$$\mu_{r,r+1;n}^{(k_1,k_2)} = \frac{n!}{(r-1)!(n-r-1)!}\frac{4\theta^2}{U-V}\int_v^u\int_x^u x^{k_1-3}y^{k_2-3}e^{-\theta/y^2}e^{-\theta/x^2}$$

$$\left(e^{-\theta/x^2} - V\right)^{r-1}\left(U - e^{-\theta/y^2}\right)^{n-r-1} dydx \tag{2.20}$$

or by using the definition of g(x) and G(x), equation (2.20) can be written as follows

$$\mu_{r,r+1;n}^{(k_1,k_2)} = \frac{n!}{(r-1)!(n-r-1)!}\int_v^u x^{k_1} g(x)[G(x)]^{r-1} I(x)dx, \tag{2.21}$$

where

$$I(x) = \int_x^u y^{k_2}[1-G(y)]^{n-r-1} g(y)dy, \tag{2.22}$$

from (1.8) and (2.22), we get

$$I(x) = 2\theta\int_x^u y^{k_2-3}[1-G(y)]^{n-r-1}[G(y)+V]dy$$

$$= 2\theta[I_1 + I_2]$$

where

$$I_1 = \frac{U}{U-V}\int_x^u y^{k_2-3}[1-G(y)]^{n-r-1} dy,$$

and

$$I_2 = \int_x^u y^{k_2-3}[1-G(y)]^{n-r} dy.$$

By using integration by parts, then we have

$$I_1 = \frac{U}{(U-V)}\cdot\frac{1}{k_2-2}\left\{-x^{k_2-2}[1-G(x)]^{n-r-1}\right.$$

$$\left. + (n-r-1)\int_x^u y^{k_2-2}g(y)[1-G(y)]^{n-r-2}dy\right\} \tag{2.23}$$

Similarly

$$I_2 = \frac{1}{k_2 - 2}\Big[-x^{k_2-2}[1-G(x)]^{n-r}$$

$$+(n-r)\int_x^u y^{k_2-2} g(y)[1-G(y)]^{n-r-1}\,dy\Big] \qquad (2.24)$$

From (2.24), (2.23), (2.22) and (2.21) we have

$$\mu_{r,r+1;n}^{(k_1,k_2)} = \frac{n!}{(r-1)!(n-r-1)!} \cdot \frac{2\theta}{k_2 - 2}$$

$$\Big\{\frac{U}{U-V}(n-r-1)\int_v^u\int_x^u x^{k_1} y^{k_2-2} g(x)g(y)[G(x)]^{r-1}[1-G(x)]^{n-r-2}\,dydx$$

$$-(n-r)\int_v^u\int_x^u x^{k_1} y^{k_2-2} g(x)g(y)[G(x)]^{r-1}[1-G(x)]^{n-r-1}\,dydx$$

$$-\frac{V}{U-V}\int_v^u x^{k_1+k_2-2} g(x)[G(x)]^{r-1}[1-G(x)]^{n-r-1}\,dx$$

$$-\int_v^u x^{k_1+k_2-2} g(x)[G(x)]^{r}[1-G(x)]^{n-r-1}\,dx\Big\} \qquad (2.25)$$

Simplifying (2.25) we get (2.19) which prove Theorem (4).

3. CONCLUSION AND REMARKS

The recurrence relations for the single and product moments of order statistics are established in both truncated and non-truncated IR distributions. These relations may be used to compute the single and product moments in a simple recursive manner for any sample size. Also we have the following remarks:

1- Setting $y = \sqrt{x}$ in (2.4), (2.5) and (2.15) we obtain the corresponding recurrence relations for single and product moments of order statistics in the case of inverse exponential distribution (Lin, Duran and Lewis, 1989).

2 - If $v = 0$ and $u = \infty$, then (2.15) reduces to (2.4).

3 - If $v = 0$ and $u = \infty$, then (2.21) reduces to (2.5).

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